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ENHANCED FLUCTUATIONS IN PLASMAS

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1. INTRODUCTION

Most plasmas, whether they are produced in the laboratory or naturally occurring, are far from thermal equilibrium. One consequence of this is that they are very "noisy", i.e., various modes of the plasma are highly excited to more than the energy of KT/2 per propagating mode appropriate to thermal equilibrium. This excited wave spectrum may take the form of macroscopic turbulence excited externally or by instabilities, or at a lower amplitude level is continually present in the form of the natural fluctuations of a many-body system. Such turbulent motion plays an important role in the processes of scattering and emission of electromagnetic radiation by a plasma, and also contributes to transport processes such as the rapid diffusion of plasma across a magnetic field through wave-particle scattering.

The chart below illustrates the domains of theoretical activity applicable to several amplitude levels of disordered plasma motion. The conventional BBGKY[1] theory is in the second box. It is useful for plasmas in which $\Delta^{-1} \equiv n_o L_D^3 >> 1$ where n_o is the average electron density and L_D the Debye length, but it has only been practical to calculate the fluctuation spectrum to $O(n_o L_D^3)^{-1}$ for cases in which the one particle distribution, f, is nearly spatially homogeneous and stable. Plasmas which are "weakly unstable" and for which effects of $O(n_o L_D^3)^{-1}$ can be neglected fall in box 4, i.e., the domain of weakly turbulent Vlasov plasmas to which perturbation techniques (e.g. the quasi-linear theory) of one sort or another can be applied.

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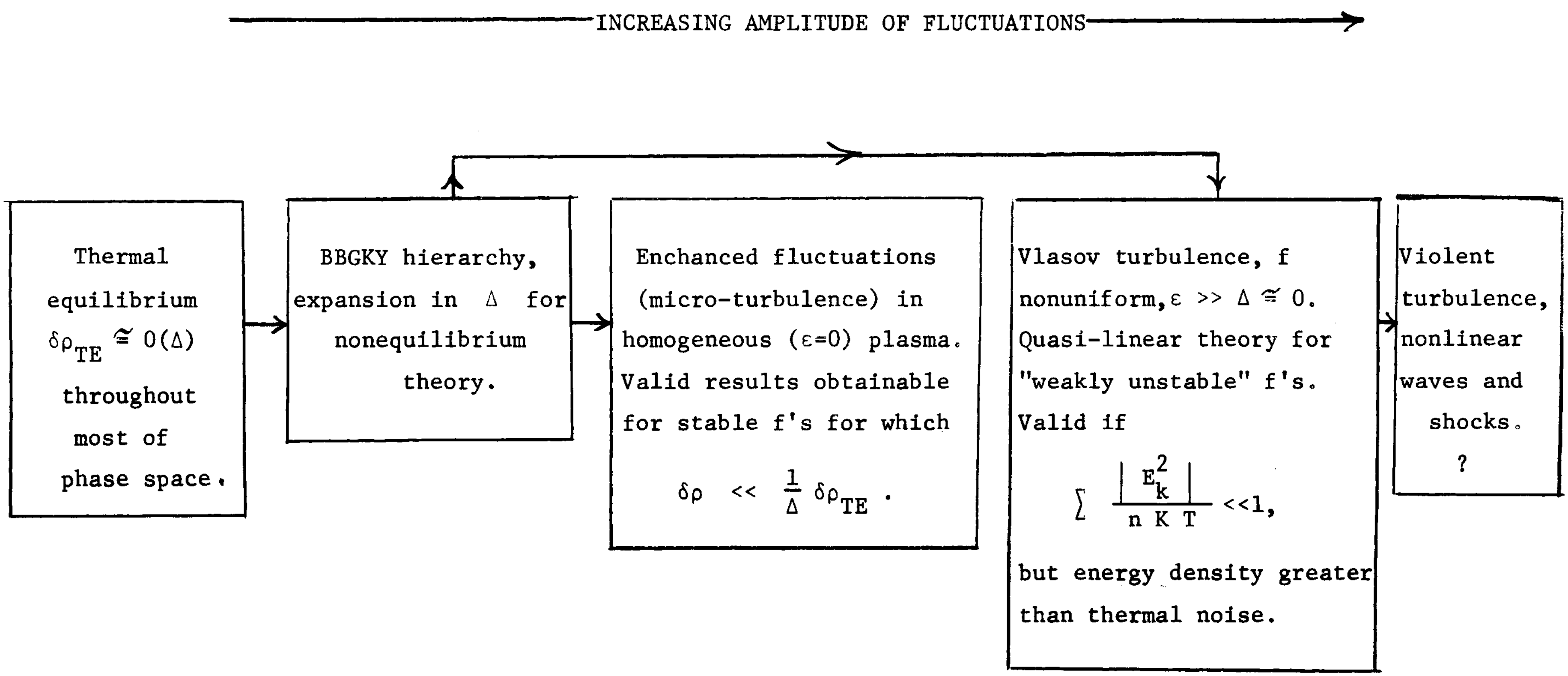

$\Delta = (n_o L_D^3)^{-1}$ = plasma discreteness parameter,

$\varepsilon \cong O(\delta f/f)$.

In this paper we confine our discussion to the domain of box 3, i.e., to some simple comments on enhanced fluctuations excited by non-maxwellian but stable plasmas.

2. ELECTRIC FIELD FLUCTUATIONS OF A NONTHERMAL PLASMA

Consider the electric field fluctuations in a Coulomb gas of particles free of magnetic field. We define the auto correlation function $\underline{\underline{C}}$ and spectral density $\underline{\underline{S}}$ for these fluctuations by,

$$\underline{\underline{C}}(\underline{x},t) = \left\langle \underline{E}(\underline{X},T)\ \underline{E}(\underline{X}+\underline{x},T+t)\right\rangle , \tag{1}$$

$$\underline{\underline{S}}(\underline{k},\omega) = \int_{-\infty}^{\infty} d\underline{x}\ dt\ e^{-i(\underline{k}\cdot\underline{x}+\omega t)}\ \underline{\underline{C}}(\underline{x},t) \quad . \tag{2}$$

$\underline{\underline{C}}$ measures the relatedness at different points of a fluctuating quantity and $\underline{\underline{S}}$ gives the Fourier spectrum of the fluctuations. $\left[\underline{\underline{C}}(|x|\to\infty \text{ or } t\to\infty)\to 0\right]$. Formulas for bremsstrahlung, or radiation scattering cross-sections, etc., can be expressed in terms of various spectral densities like $\underline{\underline{S}}$.

Now it has been shown by Rostoker(2) and simplified more recently by Dawson and Nakagawa(3) that many results of kinetic theory can be obtained by a consideration of test-particle results. In particular it is simple to calculate the spectral density $\underline{\underline{S}}$ by using the result that to first-order in Δ the plasma can be regarded as an uncorrelated gas of dressed test-particles. By a dressed test-particle we mean a particle plus its attendant disturbance of the surrounding medium, i.e., polarization cloud and Cerenkov wake. Thus the electric field at $\underline{x}$ due to a test-particle of charge e_t, velocity $\underline{v}_1$, located at $\underline{x}_1$ at time t is,

$$\underline{E}(\underline{x};\underline{x}_1,\underline{v}_1 t) = -\frac{e_t}{2\pi^2}\int \frac{d\underline{k}\ i\underline{k}\ e^{i\underline{k}\cdot(\underline{x}-\underline{x}_1)}}{k^2\ D(\underline{k},-i\underline{k}\cdot\underline{v}_1)}$$

where $D(\underline{k},i\omega)$ is the plasma dielectric constant ,

$$D(\underline{k},i\omega) = 1 - \sum \frac{\omega_j^2}{k^2} \int \frac{d\underline{v}\ \underline{k} \cdot \partial f_j/\partial \underline{v}}{(\omega + \underline{k}\cdot\underline{v})} \quad , \quad \mathrm{Im}(\omega) < 0 \quad , \tag{4}$$

and $\omega_j = (4\pi n_{oj} e_j^2/m_j)^{\frac{1}{2}}$ the plasma frequency of the j^{th} species of particle in the plasma.

We next calculate $\underline{\underline{C}}$ as follows:

$$\left\langle \underline{E}(\underline{x},t)\ \underline{E}(\underline{x}',t') \right\rangle$$

$$= \sum \frac{n_{oj}}{V} \int E(\underline{x};\underline{x}_1,\underline{v}_1,t)\ \underline{E}(\underline{x}';\underline{x}_1',\underline{v}_1',t')\ W(\underline{x}_1,\underline{v}_1,t;\underline{x}_1',\underline{v}_1',t')$$

$$d\underline{x}_1\ d\underline{v}_1\ d\underline{x}_1'\ d\underline{v}_1'$$

where W is the joint probability of finding a particle in $d\underline{x}_1\ d\underline{v}_1$ at t and the same particle in $d\underline{x}_1'\ d\underline{v}_1'$ at t'. There are n_{oj} such contributions to $\langle\ \rangle$ from the j^{th} species. V is a volume in which the plasma is enclosed and over which the space integrations go, and we take $V \to \infty$ later.

Now if the test particles are assumed to be uncorrelated,

$$W = V\ f(\underline{v}_1) \quad \delta\left[\underline{x}_1' - \underline{x}_1 + \underline{v}_1(t'-t)\right]\ \delta(\underline{v}_1'-\underline{v}_1) \quad ,$$

which leads immediately to,

$$\underline{\underline{S}}(\underline{k},\omega) = (4\pi e)^2 \frac{\underline{kk}}{k^4} \frac{2\pi}{k} \frac{\sum n_{oj} F_j(\frac{\omega}{k})}{|D(\underline{k},i\omega)|^2} \quad , \tag{5}$$

$$F_j(u) = \int d\underline{v} \quad \delta(u - \frac{\underline{k}\cdot\underline{v}}{k}) \quad f_j(\underline{v}) \ . \tag{6}$$

In figure 1 we have drawn the function $S = T_r(\underline{\underline{S}})$ as a function of ω for a fixed wavenumber $k < k_D$, and for the case of thermal equilibrium, i.e., maxwellian $f's$. The spectral density S has two shelves at $0 < \omega \lesssim O(kV_i)$ and for $\omega \lesssim O(kV_e)$. These correspond to Fourier components which drift with phase velocities of order the thermal velocities V_i and V_e of the ions and electrons respectively. There is also a sharp resonance at $\omega \cong \omega_e$ and width $\gamma_L(\frac{\omega_e}{k})$ which is the Landau damping decrement for longitudinal plasma oscillations of phase velocity ω_e/k. This resonance becomes wider and has more area under it for electron distributions which have non maxwellian tails as illustrated by distribution (ii). It has its origin in the emission of Cerenkov electron plasma waves by the high energy electrons in the tail of the distribution and their subsequent reabsorption through Landau damping.

The energy density in the propagating $(k < k_D)$ spectrum of enhanced plasma oscillations represented by the resonance in S at $\omega \cong \omega_e$ can be calculated from (5) as,

$$\left.\frac{\langle E^2\rangle}{8\pi}\right|_{k<k_D} \cong \frac{8n_o e^2}{\omega_e} \int_o^{k_D} k\, dk \, \frac{F_e(\frac{\omega_e}{k})}{\left| F'_e(\frac{\omega_e}{k})\right|} \tag{7}$$

for an electron ion plasma with $n_{oe} = n_{oi} = n_o$. Consider as an example of a distribution of electrons with a high-energy tail the function

$$f_e = \frac{\beta}{(2\pi)^{3/2} V_e^3} \exp\left(-\frac{v^2}{2V_e^2}\right) + \frac{(1-\beta)}{(2\pi)^{3/2} V_E^3} \exp\left(-\frac{v^2}{2V_E^2}\right), \tag{8}$$

where $1 \gg (1-\beta) \geq 0$ and $V_E^2 \gg V_e^2$. The case $\beta = 1$ recovers thermal equilibrium. One can then readily show ,

$$\frac{\langle E^2\rangle}{\langle E^2\rangle_{\text{Thermal Eq.}}} \cong \left(\frac{V_E^2}{V_e^2}\right) \Big/ \ln\left[\frac{V_E}{V_e(1-\beta)}\right]$$

Thus for example if $V_E \cong 20V_e$,$(1-\beta) \cong .1$, we have an enhancement factor of 100 in the energy density of these fluctuations compared to thermal equilibrium.

3. EMISSION AND SCATTERING OF RADIATION

Such enhanced fluctuations naturally lead to enhanced emission(4) or scattering(5) of electromagnetic radiation by a plasma. For an optically thin slab of maxwellian plasma the bremsstrahlung emission is largely continuous with two sharp peaks with negligible area under them near frequencies ω_e and $2\omega_e$ (see figure 2, case (i)). These peaks have their origin in the emission that takes place in the combination scattering of ion plasma oscillations (ipo) by electron plasma oscillations (epo) , and the latter waves by themselves. Thus,

$$\begin{aligned} &(\text{epo}) + (\text{ipo}) \rightarrow \text{radiation at } \omega_e , \\ &(\text{epo}) + (\text{epo}) \rightarrow \text{radiation at } 2\omega_e . \end{aligned} \tag{10}$$

The emission at these two frequencies is superposed on the background continuous spectrum and is given by ,(4)

$$I_{\omega_e} \cong \frac{e^2\omega_e^2\sqrt{3}\, V_e}{3\pi^2 c^3} \int_0^{k_D} k^2\, dk \, \frac{F_e(\frac{\omega_e}{k})}{\left|F_e'(\frac{\omega_e}{k})\right|} , \tag{11}$$

$$I_{2\omega_e} \cong \frac{\omega_e^4 \sqrt{3}\, e^2}{5\pi^2 c^5} \int_0^{k_D} dk \frac{F_e^2(\frac{\omega_e}{k})}{\left| F_e'(\frac{\omega_e}{k}) \right|^2} \text{ ergs/sec/cm}^3 \, . \tag{11}$$

For the distribution (8) these become ,

$$\begin{aligned} I_{\omega_e} &\cong 6.\ 10^{-25} n_o^{5/2} T^{-3/2} \left(\frac{V_E}{c}\right)^2 \alpha^{-4} \\ I_{2\omega_e} &\cong 5.\ 10^{-25} n_o^{5/2} T^{-3/2} \left(\frac{V_E}{c}\right)^4 \alpha^{-3} \quad \text{ergs/sec/cm}^3 \end{aligned} \tag{12}$$

with $\alpha^2 = 2\ell n\left[V_E/V_e(1-\beta)\right]$. We recover the thermal equilibrium result by setting $\alpha = 1$, $V_E = V_e$ in the above expressions. Similar formulas for enhanced scattering cross-sections of radiation also exist.

In conclusion it is interesting to point out that enhanced fluctuations produced by suprathermal electrons play an important role in a number of astrophysical plasma phenomena, Types II and III radio bursts from the Sun have a two-harmonic structure at the plasma frequency ω_e and $2\omega_e$ of the coronal plasma, and can be satisfactorily understood[(6)] in terms of the enhanced emission of the type we have discussed. Another example is the high-energy tail of photoelectrons for the ionospheric plasma. This gives rise to an enhanced cross-section for backscatter of an incident radio wave of frequency Ω at frequencies $\Omega \pm \omega_e$. This has recently been observed by Perkins, Salpeter, and Yngvesson[(5)] using the Arecibo radio dish.

ACKNOWLEDGEMENT

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FIGURE CAPTIONS

Figure 1

Plot of spectral density for a maxwellian distribution (case (i)) and a distribution with a nonmaxwellian tail (case (ii)).

Figure 2

Schematic plot of emission intensity of radiation for the maxwellian and nonmaxwellian distributions (i) and (ii) .

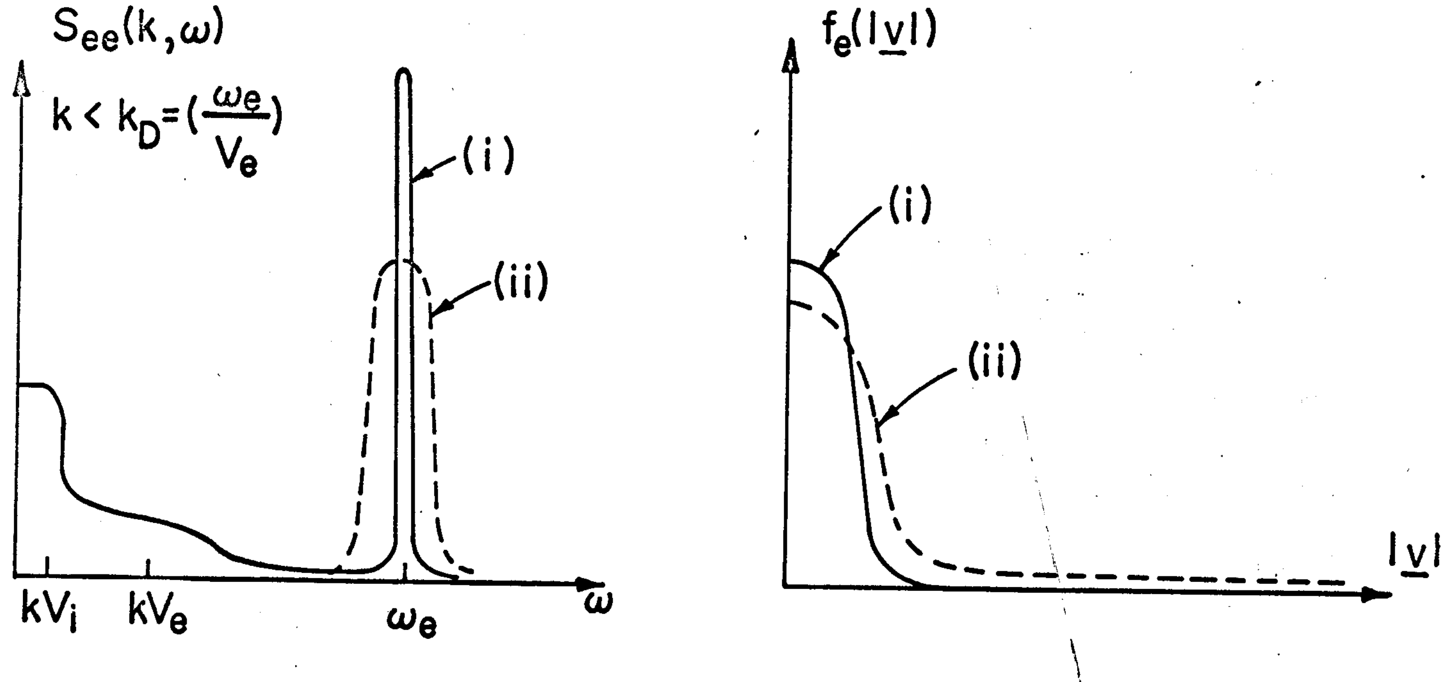
$S_{ee}(k,\omega)$
$k < k_D = (\frac{\omega_e}{V_e})$
(i)
(ii)
kV_i
kV_e
ω_e
ω
$f_e(|\underline{v}|)$
(i)
(ii)
$|\underline{v}|$

Figure 1

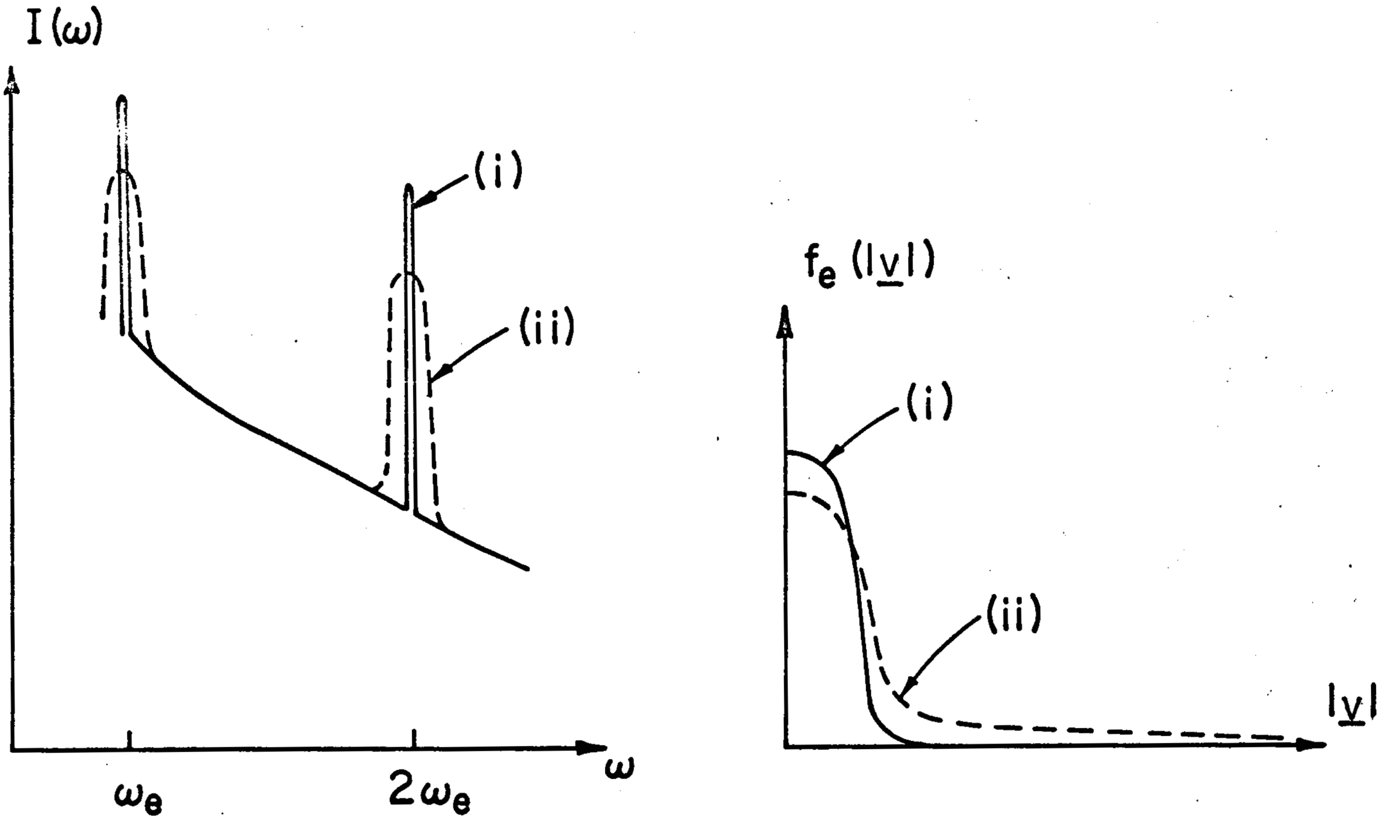

Figure 2